Lesson 1

NUMBER SYSTEM

LEARNING IN THIS CHAPTER

- Number Systems - Decimal, Binary, Octal, Hexadecimal
- Conversion of Decimal into Binary Number System
- Binary to Decimal Number System
- Computer Arithmetic
- Binary Addition, Subtraction, Multiplication and Division

In early days when there were no means of counting, people used to count with the help of fingers, stones, pebbles sticks, etc. These methods were not adequate and had many limitations. Many number systems were introduced with the passage of time, like:

- Decimal number system
- Octal number system
- Binary number system
- Hexadecimal number system

DECIMAL NUMBER SYSTEM

The need for counting has paved the path to introduce Decimal number system in which 0,1,2,3...9 are used to form any number. Most of our arithmetic operations are performed with decimal numbers.

It consists of ten digits i.e., 0 to 9 with the base 10. Each number can be used individually or they can be grouped to form a numeric value. E.g., 82, -256, 52.87 etc. The value of each digit in a number depends upon the following:

- The face value of the digits
- The base of the number system
- The position of the digit in the number

For example, the number 547 can be understood in powers of its base:

Example

5 4 7

$7 * 10^0$ → units = 7

$4 * 10^1$ → tens = 40

$5 * 10^2$ → hundreds = 500

547

OBSERVATION

The positional value of each digit increases ten folds as we move from right to left. In the above mentioned example; 5, 4 and 7 are the face values of digits and their place values are hundreds, tens and units respectively. The face value depends on the position of the digit in the number.

BINARY NUMBER SYSTEM

The Binary number system consists of only two digits, i.e., zero and one (0 and 1). Since this system uses two digits, it has the base 2. All digital computers use this number system and convert the data input from the decimal format into its binary equivalent.

WHY BINARY?

A computer cannot understand human language. The data, which is entered into a computer is converted into binary form because a computer understands only binary code. It further converts the binary results into their decimal equivalents for output.

CONVERSION OF DECIMAL INTO BINARY NUMBER SYSTEM

The equivalence between binary and decimal numbers can be understood with the given examples. To convert a decimal number into a binary number, follow the given rules:

- Divide the given decimal number with the base 2.
- Write down the remainder and divide the quotient again by 2.
- Repeat the step 2 till the quotient is zero.

Let us understand the conversion of Decimal number into Binary number with the given examples:


Example

Example 1:

2	25	
2	12	1 → Least Significant Digit
2	6	0
2	3	0
2	1	1
	0	1 → Most Significant Digit

Thus $(25)_{10} = (11001)_2$

The base of number is given as subscript.

Example 2:

2	321	
2	160	1 → Least Significant Digit
2	80	0
2	40	0
2	20	0
2	10	0
2	5	0
2	2	1
2	1	0
	0	1 → Most Significant Digit

Thus $(321)_{10} = (101000001)_2$

Remainders, which are obtained in each step are written in reverse order, to form the binary equivalent of a decimal number.

BINARY TO DECIMAL NUMBER

To convert a binary number into decimal number, follow the given steps:

- Multiply each binary number with its positional value, which is in terms of powers of 2, starting from the extreme right digit.
- Increase the power one by one, keeping the base fixed as 2.
- Sum up all products to get the decimal number.

Fact File

Aryabhat was India's greatest mathematician and astronomer. He introduced the concept of 0 (zero) without which modern computer technology would have been non-existent.

Let's Know More

Base or Radix of a Number System

The base of the number system is the number of digits used in it. E.g., Since the decimal number system uses 10 digits, its base is 10.

Fact File

Gottfried Leibniz, a German mathematician is credited with the invention of the modern Binary number system.

Let Us Recall

Which number system do we use?

Example 1:

THTU

$(1010)_2$

0×2^0 - Units = 0

1×2^1 - Tens = 2

0×2^2 - Hundreds = 0

1×2^3 - Thousands = 8

Thus $(1010)_2 = (10)_{10}$

Example 2:

$(1\ 0\ 0\ 1)_2$

$1 \times 2^0 = 1$

$0 \times 2^1 = 0$

$0 \times 2^2 = 0$

$1 \times 2^3 = 8$

Thus $(1001)_2 = (9)_{10}$

Example 3:

$(110001001)_2 = 1 \times 2^8 + 1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 256 + 128 + 0 + 0 + 0 + 8 + 0 + 0 + 1$

$= 393$

Thus $(110001001)_2 = (393)_{10}$

OCTAL NUMBER SYSTEM

The Octal number system consists of 8 digits i.e., 0 to 7 with the base 8. The procedure of octal to decimal conversion is similar to 'binary to decimal' conversion. The only difference is the change of base.

HEXADECIMAL NUMBER SYSTEM

This number system contains 16 digits, and therefore has the base 16. It uses the digits (0-9) and (A-F).

COMPUTER ARITHMETIC

As a computer understands only binary code, the data given by the user is converted into binary code for processing. This processing may involve various kinds of arithmetic operations, such as addition, subtraction, multiplication, division, etc. on binary numbers.

BINARY ADDITION

The technique used to add binary numbers inside the computer is very easy and simple. This is performed in the same way as you perform addition with decimal numbers. The following table illustrates the addition of two binary digits:

a	b	a + b = c
0	0	0 + 0 = 0
0	1	0 + 1 = 1
1	0	1 + 0 = 1
1	1	1 + 1 = 10

While adding 1 + 1, the output will be 10, where 0 is written under the same column and carry over 1 is shifted to the next place as it happens in decimal number addition.


Example

Example 1:

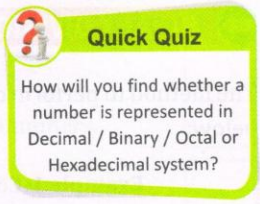
Compute $(1000)_2 + (111)_2$

$$\begin{array}{r} 1000 \\ +\ 0111 \\ \hline 1111 \end{array}$$

Example 2:

Compute $(11111)_2 + (1011)_2$

$$\begin{array}{r} \textbf{1111}\quad \textbf{Carry over} \\ 11111 \\ +\ 01011 \\ \hline 101010 \end{array}$$


Quick Quiz

How will you find whether a number is represented in Decimal / Binary / Octal or Hexadecimal system?

BINARY SUBTRACTION

The rules given in the table must be followed to perform binary subtraction:

NOTE

The number is borrowed when 1 is subtracted from 0(10 -1 = 1).

a	b	a - b = c
0	0	0 - 0 = 0
1	0	1 - 0 = 1
1	1	1 - 1 = 0
0	1	0 - 1 = 1

[with a borrow taken from next place] i.e.,10-1=1


Example

Example 1:

Compute $(1111)_2 - (1010)_2$

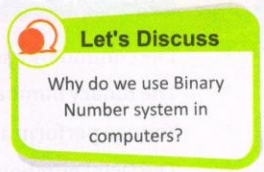
$$\begin{array}{r} 1\ 1\ 1\ 1 \\ -\ 1\ 0\ 1\ 0 \\ \hline 0\ 1\ 0\ 1 \end{array}$$

Example 2:

Compute $(1100)_2 - (11)_2$

Borrowed 1 (Balance 0) → 10 (Balance 1); Again Borrowed 1 → Number is now 10 $\frac{-1}{1}$

$$\begin{array}{r} 1\quad 1\quad 0\quad 0 \\ -\ 0\quad 0\quad 1\quad 1 \\ \hline 1\quad 0\quad 0\quad 1 \end{array}$$


Let's Discuss

Why do we use Binary Number system in computers?

BINARY MULTIPLICATION

The rules for performing multiplication using binary numbers is same as that of decimal numbers. The given table illustrates the multiplication of two binary digits:

a	b	a * b = c
0	0	0 * 0 = 0
0	1	0 * 1 = 0
1	0	1 * 0 = 0
1	1	1 * 1 = 1


Example

Example 1:

Compute $(101)_2 \times (11)_2$

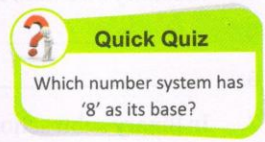
$$\begin{array}{r} 101 \\ \times\ 11 \\ \hline 101 \\ +\ 101\times \\ \hline 1111 \end{array}$$

Sum =

Example 2:

Compute $(1111)_2 \times (101)_2$

$$\begin{array}{r} 1111 \\ \times\ 101 \\ \hline 1111 \\ 0000\times \\ +\ 1111\times\times \\ \hline 1001011 \end{array}$$


Quick Quiz

Which number system has '8' as its base?

BINARY DIVISION

The method to perform division of two binary numbers is same as that of decimal numbers. See the example given below:

Example 1: Compute $(110)_2 \div (10)_2$

```
    11   <- Quotient
10 | 110  <- Dividend
     10
     ---
     010
      10
      --
      00   <- Remainder
```

Example 2: Compute $(10000111)_2 \div (1001)_2$

```
    01111   <- Quotient
1001 | 10000111  <- Dividend
       1001
       ----
       001111
        1001
        ----
        01101
         1001
         ----
         01001
          1001
          ----
          0000   <- Remainder
```

Recap of the Chapter

- The commonly used number system is Decimal number system with the base 10.
- The Binary number system consists of two digits i.e., 0 and 1 and has the base 2.
- You can perform arithmetical operations on binary numbers.
- The Octal number has the base 8.
- The Hexadecimal number system has the base 16.

Brain DEVELOPER

A. Fill in the blanks:

1. The base of Binary number system is .....
2. The base of ..... system is 10.
3. Octal Number system consists of ..... digits.
4. In Binary addition, 1+1 equals to .....
5. ..... number system is understood by the computer system.
6. ..... *uses 16 symbols to represent numbers.*
7. In Binary subtraction, 1-1 equals .....

HINTS

- 0
- Binary
- Decimal number
- Hexadecimal
- 2
- 8
- 10

B. State True or False:

1. The decimal number system consists of 10 digits i.e., 0 to 9. ☐
2. The method to perform division of two binary numbers is not same as that of decimal numbers. ☐
3. 1 multiplied by 0 equals to 0. ☐
4. Charles Babbage introduced the concept of 0 (Zero). ☐
5. The numbers used in Octal number system are 1 to 7. ☐

C. Application Based Questions:

1. Ratika's computer teacher asked her to convert the Octal number to Decimal number. Suggest her the method which she should apply in converting the Octal number.

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2. The teacher has given an assignment to Saurabh on Binary subtraction. Saurabh is confused how to subtract 1 from 0. Help him in solving the problem.

.....

D. Multiple Choice Questions:

1. ..... introduced the concept of 0 (Zero).

   a. Ada Lovelace b. Aryabhat c. Bill Gates

2. A ..... converts the decimal format into its binary equivalent.

   a. Digital Computer b. Cell Phone c. Abacus

3. A computer understands only ..... code.

   a. English b. French c. Binary

4. In Binary multiplication, 1x1equals to .....

   a. 0 b. 1 c. 2

5. To convert Decimal number into Binary number, divide the number by .....

   a. 2 b. 8 c. 10

E. Answer the following:

1. Explain Number system and its commonly used types.

.....

.....

.....

D. Find the difference between the following Binary numbers:

a. 10011 - 01010	b. 11001001 - 01100110	c. 111 - 001

E. Multiply the following Binary numbers:

a. 101 x 011	b. 1011 x 101	c. 101010 x 1011

F. Divide these Binary numbers:

a. 1111 ÷ 11	b. 111001 ÷ 101	c. 111111111 ÷ 1011

GROUP DISCUSSION — For Concept Clarity

Divide the class into two groups and discuss on the topic:
Decimal Number System vs Binary Number System

PROJECT WORK — Using Creativity

Make a presentation on **Number System**. Set a beautiful background. Apply nice formatting and animation effects on it.

ONLINE LINKS — Looking For More

To learn more about Number System, visit the following sites:

- http://209.184.112.198/~gboswell/net1305102102/Computer_Number_Systems.pdf
- http://www.thevbprogrammer.com/Ch04/Number%20Systems%20Tutorial.pdf